International Journal of Academic Research and Development
www.academicjournal.in
ISSN: 2455-4197, Impact Factor: RJIF 5.22
Received: 27-12-2020, Accepted: 15-01-2021, Published: 27-01-2021
Volume 6, Issue 1, 2021, Page No. 62-66

Quantification in effective viscosity of fluid in a porous medium by brinkman equation

Kesar Singh[1], CV Gupta[1], RB Singh[2]
[1] Department of Mathematics, B.S.A. College, Mathura, Uttar Pradesh, India
[2] Scientist (UGC), Department of Zoology, School of Life Sciences, Dr. Bhimrao Ambedkar University, Khandari Campus, Agra, Uttar Pradesh, India

Abstract
The quantification of Brinkman's effective viscosity, which arises due to the presence of the porous medium and its effect on increasing or decreasing the fluid viscosity in relation to the base fluid viscosity. In present investigation is motivated in the main part by the lack of consensus in the available literature on the ranges of effective viscosity. This work provides a model of quantifying the effective viscosity by incorporating the porous microstructure in the volume-averaged Navier-Stokes equations. Extensive analysis and testing are provided in the present work which considers three different porous microstructures.

Keywords: effective viscosity, brinkman equation, porous medium

Introduction

In spite of the popularity of Darcy's law in the study of groundwater flow, it suffers some limitations that include not accounting for microscopic inertia that arises due to tortuosity of the flow path [1]. Darcy's law [2] has been argued to be valid in low permeability, low porosity media where variations at the microscopic, pore-level length scale are negligible at the macroscopic length scale.

Brinkman [3] introduced an extension to Darcy's law [2] by including a viscous term to account for the viscous shear effects that are important in the thin boundary layer near a macroscopic boundary. Accordingly, equations governing the steady flow of a Newtonian fluid through a porous medium composed of a swarm of particles fixed in space are given by the following equation of continuity, and linear momentum equation that is due to Brinkman equation [3].

Conservation of Momentum (Brinkman Equation):

$$-\nabla p + \mu\nabla^2 \bar{u} - \frac{\mu}{k}\bar{u} = 0 \quad \text{….. (1)}$$

Where $\bar{u}$ is the ensemble-averaged velocity, μ is the base fluid viscosity, μ is the viscosity of the fluid saturating the porous medium, k is the permeability and p is the pressure. The term $\frac{\mu}{k}\bar{u}$ is the Darcy resistance, and the term $\mu\nabla^2 \bar{u}$ is the viscous shear term that facilitates imposition of a no-slip velocity condition on solid boundaries. It is clear that when $\mu = 0$, Brinkman's equation (3) reduces to Darcy's law [2].

It has been long realized and documented that there is an increased effective shear viscosity in constricted geometries and close solid surfaces [4] and theoretically investigations that provide estimate for $\lambda = \frac{\mu^*}{\mu}$ from less than unity to as high as 10 and other authors [5, 6, 7] believe that the effective dynamic viscosity arises due to volume averaging and provided detailed theoretical analysis of its quantification. Alas; the reported studies provide no definite answer as to what the value of λ should be. However, many agree that the presence of solid walls bounding the porous flow domain, and the geometric configurations of the porous material play an important part in the value of λ, A number of studies have investigated the validity of Brinkman's equation and a number of excellent and sophisticated analyses have been carried out to quantify the effective viscosity, μ . Some argue that the viscosity factor, or the ratio, λ. could be greater than unity or less than unity [8] and the references there in, or could be taken as unity as favoured by Brinkman[3]. Brinkman initially used Einstein's formula for the viscosity of suspension [9], to relate fluid viscosity and the effective viscosity.

$$\mu = [1 + \tfrac{5}{2}(1 - \phi)]\mu \quad \text{… (2)}$$

Where ϕ is the effective porosity of the Medium.

Many investigators have attempted to resolve the question of effective viscosity. Some argue that equation (4) is valid for high porosity media. As porosity of the medium decreases, viscosity of the saturating fluid increases. Indeed, the literature includes studies and experiments that report viscosity factors ranging from less than unity to approximately 10[6,7]. In a recent work, Breugem[8] provided elegant and thorough analysis of the effective viscosity of fluid in a channel-type porous medium and provided the following effective viscosity estimate, referred to here as Breugem's Estimate:

$$\mu = \begin{Bmatrix} \frac{1}{2}\left(\varphi - \frac{3}{7}\right)\mu; \varphi \geq \frac{3}{7} \\ 0; \varphi < \frac{3}{7} \end{Bmatrix} \quad \text{…. (3)}$$

Hence, the viscosity factor takes the form:

$$\lambda = \frac{\mu}{\mu} = \frac{1}{2}\left(\varphi - \frac{3}{7}\right). \quad \text{…… (4)}$$

The effects of bounding solid walls on slow flow over regular, square arrays of circular cylinders between two parallel plates. Their results indicate that, between the two limits of the Darcian porous medium and the viscous flow, the magnitude of the viscosity factor λ needs to be close to unity in order to satisfy the non-slip boundary conditions at the bounding walls [6].

It seems to be no consensus as to what the viscosity factor should be, no as to how the effective viscosity could be computed, and how it depends on the porosity of the medium. However, it is agreed upon that the effective viscosity depends on factors that include the type of fluid, the speed of the flow, porosity and permeability of the medium, and the presence of bounding solid walls to the medium.

The above lack of consensus motivates the current work in which we provide analysis of the effective viscosity in flow through porous media. This will be accomplished by analyzing a general model of flow through isotropic porous media that was developed [1, 10, 11] based on intrinsic volume averaging. We devise a method for evaluating the effective viscosity under Poiseuille flow based on analyzing the geometric factors of five different types of porous media. Expressions for the effective viscosity, mean velocity, and maximum velocity will be obtained. We also evaluate the viscous flow limit and the Darcy limit, and derive a threshold for the transition from Darcy's to Brinkman's flow regimes. Our goal is to analyze the effect of porosity function on the effective viscosity.

Materials and Methods

A number of authors have derived equations of flow through porous media by averaging the Navier-Stokes equations over a representative elementary volume, REV, and quantifying the frictional forces exerted by the porous medium on the traversing fluid through an idealized description of the porous microstructure [1, 9, 10]. Letting V be the bulk volume of an REV (that is, the combined volume of the solid and pore space), and V_φ the volume of pore space, then porosity of the medium is defined by

$$\varphi = \frac{V_\varphi}{V} \qquad \dots (5)$$

The Darcy resistance term can be argued to be dependent on porosity function in order to account for different types of porous structures. What gives rise to the Darcy resistance term in the process of intrinsic volume averaging is a surface or integral of the form [1].

Through their concept of representative unit cell, RUC, Du Plessis and Masliyah [1, 6], quantified the integral in with the help of a geometric porosity function. In the absence of inertial and gravitational effects, Du Plessis and Masliyah's averaged equations take the following form for constant porosity media, written in terms of the specific discharge, $\bar{q}$:

$$-\nabla p\varphi + \frac{\mu}{\varphi}\nabla^2\bar{q} - \frac{\mu}{\varphi}F\bar{q} = 0 \qquad \dots (6)$$

Where $\bar{q} = \varphi\bar{u}_\varphi$ and $\bar{u}_\varphi$ is the intrinsic averaged velocity, p_φ is the intrinsic averaged pressure, F is a porosity function that depends on the porous microstructure and tortuosity of the medium.

Brinkman's intrinsic velocity and pressure in equation (1) can thus be identified with the intrinsic velocity and pressure of equation (7). We point out that while equation (1) involves the effective viscosity, μ^*, equation (8) does not include μ^* explicitly. However, equation (7) includes a porosity function, F. Porosity function (or geometric factor) F is dependent on factors such as porous matrix (diameter of solid particles), the pore diameter, the porosity of the medium (defined in equation (6), and tortuosity, T, of the medium (defined as the ratio of the pore volume to the pore area). A number of expressions for F are available in literature.

Darcy Velocity

When the flow is of the seepage type, then it is governed by Darcy's law [2]. In this case, viscous shear effects are ignored and the no-slip on the solid plates is no longer valid. We can obtain the dimensionless Darcy velocity by setting to zero the viscous shear term, as shown in equation (8).

$$U = -\frac{P}{f} \qquad \dots (8)$$

Darcy velocity u_D is the same as the mean velocity, u_m. This is a constant, uniform velocity across the channel. Furthermore, the maximum Darcy velocity $(u_D)_{max}$ is also the mean velocity.

Navier-Stokes Velocity

If $f = 0$, that is if $k \to \infty$. Equation reduces to the Navier-Stokes flow under Poiseuille conditions. Now, Taking $f = 0$, we obtain the following Navier-Stokes solution satisfying U(-1)=U(1)=0, written in terms of the dimensional variables, wherein we denote the Navier-Stokes velocity by u N:

$$u_N = \frac{p_x}{2\mu}[h^2 - y^2] \qquad \dots\dots (9)$$

The maximum velocity, $(u_n)_{max}$. occurs at y = 0 and is given by:

$$(u_n)_{max} = \frac{-h^2 p_x}{2\mu} \qquad \dots\dots (10)$$

Volumectric flow rates and effective viscosity:

The Brinkman volumetric flow rate through the porous medium is denoted here by Q_B and defined by:

$$Q_B = \int_{-h}^{h} u_B\,dy = \int_{-h}^{h}\left\{\frac{-h^2 p_x}{f\mu}\left[1 - \frac{cosh\sqrt{fy/h}}{cosh\sqrt{f}}\right]\right\}dy = -\frac{2h^3 p_x}{f\mu}\left\{1 - \frac{tanh\sqrt{f}}{\sqrt{f}}\right\} \qquad \dots\dots (11)$$

The Darcian volumetric flow rate through the porous medium is denoted here by Q_D, and defined by:

$$Q_D = \int_{-h}^{h} u_D\,dy = \int_{-h}^{h}\left\{\frac{-h^2 p_x}{f\mu}\right\}dy = -\frac{2h^3 p_x}{f\mu} \qquad \dots\dots (12)$$

The Navier-Stokes volumetric flow rate is denoted by Q N, and given by:

$$Q_N = \int_{-h}^{h} u_N \, dy = \int_{-h}^{h} \left\{ \frac{-p_x}{2\mu} [h^2 - y^2] \right\} dy = -\frac{2h^3 p_x}{3\mu} \quad \text{.... (13)}$$

Brinkman's Effective Viscosity in Relation to Base Fluid Viscosity

In order to find the effective viscosity, we repalce μ by μ_B^* and replace Q_N in (37) by Q_B of (34). This is equivalent to saying: If the Navier-Stokes volumetric flow rate Q_N across the channel is replaced by the Brinkman volumetric flow rate under the same driving pressure gradient and channel depth, what is the corresponding fluid viscosity, μ_B^*?

We thus have:

$$\mu_B^* = -\frac{2h^3 p_x}{3Q_B} = \frac{\mu f}{3 - 3\frac{\tanh\sqrt{f}}{\sqrt{f}}} = \frac{\mu f^{3/2}}{3\sqrt{f - 3\tanh\sqrt{f}}} \quad \text{.... (14)}$$

Brinkman's viscosity factor, λ_{BN}, with respect to Navier-Stokes base fluid is defined as:

$$\mu_{BN}^* = -\frac{\mu_B^*}{\mu} = \frac{f}{3 - 3\frac{\tanh\sqrt{f}}{\sqrt{f}}} = \frac{f^{3/2}}{3\sqrt{f - 3\tanh\sqrt{f}}} \quad \text{.... (15)}$$

Darcy's Effective Viscosity in Relation to Base Fluid Viscosity

In order to fing Darcy's effective viscosity, μ_D^* we replace μ by μ_D^* and replace Q_N in (15) by Q_D of (16)

$$\mu_D^* = \frac{2h^3 p_x}{3Q_D} = \frac{f\mu}{3} \quad \text{... (16)}$$

Darcy's viscosity factor, λ_{DN}. An expression for the Darcy pressure gradient is obtained and is of the form in equation (17):

$$p_x = \frac{\mu_D \mu_D^*}{h^2} = -\frac{f\mu\mu_m}{3h^2} \quad \text{.... (17)}$$

Flow limits and mean velocities

Viscous flow limit:

Viscous flow limit corresponds to λ_{BN}=1, $\mu_B^* = \mu$. It is reached when $f \to 0$, or when the permeability approaches infinity, $k \to \infty$, when $f \to 0$, the flow approaches the Navier-Stokes flow and the viscous flow limit is reached. In this case, as $f \to 0$, $\tanh\sqrt{f} \to \sqrt{f} - \frac{f\sqrt{f}}{3}$ and

$$\frac{f^{3/2}}{3\sqrt{f - 3\tanh\sqrt{f}}} \to \frac{f^{3/2}}{3\sqrt{f - 3\left(\sqrt{f} - \frac{f\sqrt{f}}{3}\right)}} = 1 \quad \text{.... (17)}$$

Darcian flow limit

The Darcian flow limit corresponds to λ_{BN}=0, μ^* =0 It is reached when the permealblity is small, or $k \to 0$. This corresponds to large values of f. The pressure gradient term can be written as:

$$-\frac{h^2 p_x}{\mu u_m} = \frac{f\sqrt{f}}{\sqrt{f - \tanh\sqrt{f}}} = \frac{f}{1 - \frac{\tanh\sqrt{f}}{\sqrt{f}}} \quad \text{.... (18)}$$

Now the Large $\sqrt{f}$, $\frac{\tanh\sqrt{f}}{\sqrt{f}} \to 0$ and the pressure gradient takes the form

$$-\frac{h^2 p_x}{\mu u_m} = f \quad \text{... (19)}$$

This Darcian flow limit depends on Darcy number (Da) φ. Since $f = h^2\frac{\varphi}{k} = \frac{\varphi}{Da} = \frac{\varphi}{K}$. Equation (18) can thus be written in the form:

$$-\frac{h^2 p_x}{\mu u_m} = \frac{\varphi}{k} \quad \text{... (20)}$$

And the Darcy permeability may be expressed in the following form:

$$k = -\frac{\varphi\mu u_m}{p_x} \quad \text{... (21)}$$

While the Darcy number can be expressed as:

$$k = Da = -\frac{\varphi\mu u_m}{h^2 p_x} \quad \text{.... (22)}$$

It is clear from equation (21) and (22) that the permeability and Darcy number, respectively, are tied to the medium properties of porosity and channel depth, the driving pressure gradient, and the fluid viscosity. However, the question of how high or low the Darcy number can be chosen in order to make Darcy's law valid, remains to be answered. In what follows we attempt to provide a partial answer in terms of the factor f.

Following Bear [4], we express the permeability in the form….

$$k = -\frac{\varphi h^3}{3} \quad \text{... (23)}$$

From equation (21) and (23) we have:

$$\frac{k}{\varphi} = -\frac{h^2}{3} = -\frac{\mu u_m}{p_x} \quad \text{... (24)}$$

Hence we obtain the following expression for the fluid viscosity:

$$\mu = \frac{h^2 p_x}{3\mu_m} \quad \text{... (25)}$$

Now, from equation (24) and the knowledge that $f = \frac{h^2\varphi}{k}$, we obtain.

$$f = \frac{h^2\varphi}{k} = \frac{3}{h^2}h^2 = 3 \quad \text{... (26)}$$

Upon using $f = 3$ in equation (16), namely, $\mu_D^* = -\frac{2h^3 p_x}{3Q_d} = \frac{f\mu}{3}$, we obtain $\mu_D^* = \mu$. The above analysis furnishes the following observation.

Observation-2: In Darcy's flow, the concept of effective viscosity is irrelevant, or the "effective viscosity" is the same as the base fluid viscosity. Furthermore, the value of $f = 3$ represents the threshold for validity of Darcy's law. With this knowledge, we can determine the value of porosity below which we can assume Darcy's law to be valid.

Mean velocity

The mean velocity in the channel is the volumetric flow rate per unit depth of the channel. That is,

$$\mu_m = \frac{Q}{2h} \quad \text{.... (27)}$$

For Navier-Stokes pressure gradient is expressed as:

$$\mu_m = \frac{Q_N}{2h}\frac{-\frac{2h^3 p_x}{3\mu}}{2h} = -\frac{h^2 p_x}{3\mu} \quad \text{... (28)}$$

and the Navier-Stokes pressure gradient is expressed as:

$$-\frac{h^2 p_x}{\mu_m} = 3\mu. \quad \text{.... (29)}$$

For porous medium and the Brinkman flow, we have:

$$\mu_m = \frac{Q_N}{2h} = \frac{h^2 p_x}{3f\mu}\left\{3 - 3\frac{\tanh\sqrt{f}}{\sqrt{f}}\right\} \quad \text{..... (30)}$$

The pressure gradient is expressed as:

$$-\frac{h^2 p_x}{\mu u_m} = \frac{3f\sqrt{f}}{\sqrt{f} - \tanh\sqrt{f}} = \frac{3f}{1 - \frac{\tanh\sqrt{f}}{\sqrt{f}}} \quad \text{.....(31)}$$

Now using equation (17), we see that as:

$$\frac{f^{3/2}}{\sqrt{f} - \tanh\sqrt{f}} \to 1, \frac{3f^{3/2}}{\sqrt{f} - \tanh\sqrt{f}} \to 3.$$

Hence $-\frac{h^2 p_x}{\mu u_m} = 3 \text{ when } \sqrt{f} \to 0$; and

$$-\frac{h^2}{u_m} p_x = 3\mu. \quad \text{..... (32)}$$

This emphasizes the fact that equation (32) reduce to equation (29) as $\sqrt{f} \to 0$, and the Brinkman flow approches the Navier-Stokes flow. The Darcian mean velocity has been obtained and is equal to the Darcy velocity, $u_D$namely:

$$\mu_m = -\frac{h^2 p_x}{f\mu} = \mu_D \quad \text{..... (33)}$$

And the Darcy pressure gradient is thus expressed as:

$$-\frac{h^2 p_x}{u_m \mu} = f \quad \text{... (34)}$$

Upon using the threshold of validity of Darcy's law, that is $f = 3$, we obtain transition to Brinkman's flow, and equation (34) yields

$$-\frac{h^2 p_x}{u_m} = 3\mu \quad \text{..... (35)}$$

The above analysis furnishes the following observations.
Observation 3: Equation (29), (32) and (35) suggest that the Brinkman pressure gradient (with appropriate scaling with respect to the relevent mean velocity and channel depth) approaches the Navier-Stokes pressure gradient when $f = 0$, while the Darcy pressure gradient approaches the Brinkman pressure gradient when $f = 3$.

Results and Discussion

Results are provide a listing of the value of $-\frac{\mu^*}{(h^2 p_x)}(\mu_D)_{max}$ for h/d=10 and h/d =20, respectively. The demonstrate the expected increase in this quantity with increasing porosity, thus indicating an increase in $(\mu_D)_{max}$with increasing porosity for a given combination of $\frac{\mu}{(h^2 p_x)}$. The maximum Darcy velocity value consistently decrease with increasing h/d. These trends persist for all geometric factors considered.
Provide a listing of the values of $-\frac{\mu}{(h^2 p_x)}(u_B)_{max}$ for h/d =10 and h/d=20, respectively. The expected increase in this quantity with increasing porosity, thus indicating an increase in $(u_B)_{max}$ with increasing porosity for a given combination of $\frac{\mu}{(h^2 p_x)}$. The maximum Brinkman velocity values consistancy decrease with increasing h/d. These trends persist for all geometric factors considered. Results obtained when using Ergun's equation are close in values to those obtained when using the Kozeny-Carmen relation.
Demonstrate an increase in this quantity with increasing porosity, thus indicating an increase in the volumetric flow rate Q B with increasing porosity for a given combination of $\frac{\mu}{(2h^3 p_x)}$. The maximum Brinkman volumetric flow rate values consistently decrease with increasing h/d. These trends persist for all geometric factors considered. Results obtained when using Ergun's equation are close in values to those obtained when using the Kozeny-Carmen relation.

The viscosity ratio $\lambda_{BN} = \frac{\mu_B^*}{\mu} = \frac{F}{3 - 3\frac{\tanh\sqrt{f}}{\sqrt{f}}}$ is illustrated for h/d =10 and h/d =20, respectively. Qualitative behavior of the viscosity ratios is demonstrate an increase in the viscosity ratio with increasing h/d, and a decrease with increasing porosity, for all geometric factors considered. When using Ergun's equation and the Kozeny-Carmen relation, the viscosity ratio gets closer and closer to unity as porosity gets closer and closer to unity (the viscous flow limit). When using other geometric factors, the viscosity ratio is still far from unity for the values of h/d tested. This points out the need to further decrease the value of h/d when these porosity functions are employed. While the case of consolidated media renders moderate results for the viscosity ratio, the value remains close to 2 even when the porosity is close to unity.

In terms of the transition from a Darcy regime to a Brinkman regime, the value of f = 3 is reached when the porosity is between 0.98 and 0.985 for the Ergun's equation and the Kozeny-Carmen relation, when h/d = 10, and a porosity between 0.991 and 0.992 for h/d = 20. For consolidated media, f = 3 is reached between the porosity values of 0.998 and 0.999, for h/d = 10. It is not reached for h/d = 20. For granular media and unidirectional fibres, f = 3 cannot be reached for the values h/d = 10 or 20. This may be an indication that for these porous structures, transition from Darcy to Brinkman regime occurs at values higher than f = 3. Furthermore, these results emphasize the importance of the geometric factors in determining the viscosity ratio and describing the Brinkman flow.

Conclusion

We have provided in this work a method of estimating the viscosity factor by Brinkman's effective viscosity based on using different geometric factors by porosity functions under Poiseuille flow. The most informative results are obtained when using Ergun's equation and the Kozeny-Carmen relation, where results indicate that the viscosity factor approaches unity as porosity approaches unity like viscous flow limit. In the analysis we have also determined a threshold value of f = 3 to serve as the transition from Darcy to Brinkman flow regime. This value corresponds to value of porosity higher than 98%, thus emphasizing that Brinkman's equation is possibly valid for high values of porosity.

References

1. Du Plessis JP, Masliyah JH. Transport in Porous Media,1991:6:207-221.
2. Darcy H. The Flow of Fluid Through Porous Media, New york, Mc Graw Hill,1856.
3. Brinkman HC. Applied Scientific Research, Soc-A,I,1947:1:27-36.
4. Cowin SC. Advances in Applied Mechanics,1974:14:279-347.
5. Francisco J, Valdes-Parada, Alberto Ochoa-Tapia and Jose Alvarez-Ramirez, Physica A,2007:385:69-79.
6. Liu H, Patil PR, Narusawa U. Entropy,2007:9:118-131.
7. Martys N, Bentz DP, Garboczi EJ, Physics of Fluids,1994:6(4):1434-1439.
8. Breugem WP. Physics of Fluids,2007:19(10):103103-10- 103104-8.
9. Einstein A, Annalender Physik,1911:34:591-592.
10. Du Plessis JP, Transport in Porous Media,1994:16:189-207.
11. Du Plessis JP, Diedericks GPJ, Computational Mechanics Publications,1997:61-104.