



## Effect of gravitational force of earth on shock waves moving in non- uniform region of sea water

\*<sup>1</sup> Rohitashw Kumar, <sup>2</sup> Mukesh Kumar

<sup>1</sup> Department of Physics, BRS-BSM (PG) College, Babrala, Sambhal, Uttar Pradesh, India

<sup>2</sup> Shivpati (PG) College, Shohrat Garh, Sidharth Nagar, Uttar Pradesh, India

### Abstract

The CCW method has been used to study the effect of gravitational force of earth on shock wave moving in non-uniform region. Assuming the initial density distributional region law as  $\rho_0 = \rho'(1 + xr)$ , where  $\rho'$  is the density of the medium,  $x$  is constant and  $r$  is propagation constant. The flow of variables behind strong shock waves propagating in non-uniform region of water. Considering the effect of gravity of earth. Neglecting the effect of overtaking disturbances, the analytical relations for shock strength (strength of tsunami), shock velocity, particle velocity, pressure and angles are obtained for freely propagation of strong diverging shock. The dependence of shock strength on gravitational force is studied. Finally, the variables are computed and discussed with the help of tables.

**Keywords:** shock waves, shock strength, gravitational force on earth

### Introduction

The study of blast wave propagation is now-a-days the target of scientist and medical doctors who are using treatment as tools. Due to its medical and technical applications, many techniques have been used to study the propagation of shock waves in water/liquids/solid/gases by researchers. The first compendium of shock wave profiles was published by Los Alamos National Laboratory (1982).

Thomas<sup>[1]</sup> used "Energy hypothesis" for spherical waves. This hypothesis was successfully applied by Bhutani<sup>[2]</sup> to cylindrical blast waves in magnetogas dynamics. A theoretical study of explosion in water (without considering the effect of gravity) is carried out by Singh and Vola<sup>[3]</sup> using energy hypothesis. Experimental verification of energy hypothesis is given by Singh *et al.*<sup>[4]</sup>, Singh<sup>[5]</sup> used energy hypothesis to explosion problem in seawater considering the effect of gravity. Vishwakarma *et al.*<sup>[6]</sup> considered earth's gravitation and time dependant energy release. The analysis is completely analytical and is used for diverging shocks only. The diverging shock propagation in uniform and non-uniform gas has been studied by Yadav<sup>[7]</sup>, Yadav and Tripathi<sup>[8]</sup> considering the effect of overtaking disturbances. These results are in good agreement with experimental results. Terao *et al.*<sup>[9]</sup> studied experimentally the spherical imploding detonation waves by a two step divergent detonation. Mazell *et al.*<sup>[10]</sup> gives a numerical study of weak shock waves propagation in a reactive bubbly liquid (water). Li and Bendor<sup>[11]</sup> gives a method C.C.W. theory for detonation waves. Ishii *et al.*<sup>[12]</sup> studied numerically and analytically the shock waves propagates through a stratified gas and found that at least three types of shock pattern can be realised asymptotically. Sacito *et al.*<sup>[13]</sup> studied experimentally and numerically the attenuation of underwater shock waves by a thin porous layer and found a reasonable agreement between the experimental and numerical results. Recently, Yadav and

Gangwar<sup>[14]</sup> have used Chester<sup>[15]</sup>, Chisnell<sup>[16]</sup>, Whitham<sup>[17]</sup> method to study the propagation of strong spherical shock in non-uniform medium neglecting the effect of overtaking disturbances. Being an important phenomenon, it is essential to take into account the effect of overtaking disturbances in the motion of shock waves. The effect of overtaking disturbances, on the propagation of shock waves in water has been studied by Yadav *et al.*<sup>[18]</sup>. Recently Yadav *et al.*<sup>[19]</sup> studied the motion of strong shock waves in a highly viscous medium in presence of overtaking disturbances and discussed the flow variables of the perturbed medium excluding temperature and entropy variation, very important variables of the medium.

The purpose of present paper is to analysis the flow variables behind strong shock wave propagating in non -uniform region of water considering the effect of gravity of earth. The Chester-Chisnell-Whitham theory is used to obtain the analytical relations for shock velocity and shock strength for freely propagation of shock. The dependence of shock strength on gravitational force is studied. The shock velocity; the pressure and fluid velocity behind the shock are discussed with the help of tables.

### Basic Equations

The basic equations governing the flow behind the shock propagating in water in presence of earth gravity are given by.

$$\frac{\partial \rho}{\partial r} + \frac{\partial}{\partial r}(\rho u) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v) + \frac{\alpha \rho u}{r} + \frac{\cot \theta}{r} \rho v = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + \frac{1}{\rho} \frac{\partial P}{\partial r} + g \cos \theta = 0 \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + \frac{1}{\rho r} \frac{\partial P}{\partial \theta} - g \sin \theta = 0 \quad (3)$$

where,  $u(r,t)$   $P(r,t)$ ,  $(r,t)$  and  $v(r,t)$  respectively the particle

velocity, pressure, density and transversal fluid velocity for the flow is the radial coordinate and is the angle made by a shock radius with vertical direction.

**Boundary Conditions**

The jump conditions are given by.

$$u = \frac{2a_0M}{(n+1)}, P = \frac{2a_0^2M^2\rho_0}{(n+1)}, \rho = \left(\frac{n+1}{n-1}\right)\rho_0$$

$$a = SMa_0\left(\frac{n-1}{n+1}\right)\rho_0, S = \sqrt{\frac{2n}{n+1}} \tag{4}$$

Where,  $M=U/a_0$  is called the Mach number,  $a_0$ ,  $P$ , and  $n$  denote the undisturbed the value of density, local sound velocity behind the shock strength, pressure the density and the specific heat index of water respectively.

**Characteristic Equations**

The characteristic form of system of equations (1) - (3) is easily obtained by forming a linear combination of two equations (1) and (2) in only direction in the (r,t) plane is-

$$dp + a\rho du + \frac{a\rho}{(u+a)}\left(\frac{au}{r} + g\cos\theta\right) dr = 0 \tag{5}$$

Equation (5) represents characteristic form of diverging shock waves. Using boundary conditions (4) in equation (5), we get-

$$M = \left[ k \frac{B g \cos\theta \rho' \left[ \frac{r^{A+1}}{(A+1)} + \frac{2x r^{A+2}}{(A+2)(S+2)} \right]}{r^A [K - \rho' r(1+xr/2)g\cos\theta]} \frac{n}{(1+xr)^{S/(2+S)}} \right]^{1/2} \tag{6}$$

Where,  $K$  is constant of integration. Equation (6) represents shock strength of diverging shock waves propagating in non- uniform medium. The shock velocity of can be written as -

$$U = a_0 \left[ k \frac{B g \cos\theta \rho' \left[ \frac{r^{A+1}}{(A+1)} + \frac{2x r^{A+2}}{(A+2)(S+2)} \right]}{r^A [K - \rho' r(1+xr/2)g\cos\theta]} \frac{n}{(1+xr)^{S/(2+S)}} \right]^{1/2} \tag{7}$$

The particle velocity and pressure just behind the diverging shock wave will be -

$$u = \frac{2a_0}{(n+1)} \left[ k \frac{B g \cos\theta \rho' \left[ \frac{r^{A+1}}{(A+1)} + \frac{2x r^{A+2}}{(A+2)(S+2)} \right]}{r^A [K - \rho' r(1+xr/2)g\cos\theta]} \frac{n}{(1+xr)^{S/(2+S)}} \right]^{1/2} \tag{8}$$

$$P = \frac{2a_0^2\rho_0}{(n+1)} \left[ k \frac{B g \cos\theta \rho' \left[ \frac{r^{A+1}}{(A+1)} + \frac{2x r^{A+2}}{(A+2)(S+2)} \right]}{r^A [K - \rho' r(1+xr/2)g\cos\theta]} \frac{n}{(1+xr)^{S/(2+S)}} \right]^{1/2} \tag{9}$$

**Results and Discussion**

Variation of shock strength with gravitational force per unit mass, propagation distance, specific heat index of water and angle -

Expression (6) represents the shock strength of spherical shock wave propagating freely in non-uniform region of water. It is found that shock strength is a function of propagation distance  $r$ , specific heat index of water  $n$ , density  $\rho$ , angle  $\cos \theta$  and force per unit mass i.e. Gravity.

Initially taking  $M=10$  at  $=2.2$  for  $n=7.5$ ,  $x=1.22$ ,  $= 3.684$  and, we have computed shock strength using equation (6) and obtained results are presented in tables (1),(2),(3)and(4).

**Table 1:** The variation of shock strength with gravitational force per unit mass i.e. Gravity

Gravity (g)	Shock strength (M)
9.62	4.1079
9.64	4.3096
9.66	4.5448
9.68	4.8241
9.70	5.1632
9.72	5.5869
9.74	6.1371
9.76	6.8927
9.78	8.0240
9.80	10.000
9.82	14.984

**Table 2:** The variation of shock strength with propagation distance

Propagation distance (r)	Shock strength (M)
2.1	2.3715
2.11	2.4761
2.12	2.5996
2.13	2.7481
2.14	2.9311
2.15	3.1635
2.16	3.4719
2.17	3.9071
2.18	4.5864
2.19	5.8678
2.2	10

**Table 3:** The variation of shock strength with specific heat index of water

Specific heat index of water (n)	Shock strength (M)
7.3	10.0296
7.4	10.014
7.5	10.000
7.6	9.9857
7.7	9.9718
7.8	9.9583
7.9	9.9452
8.0	9.9323
8.1	9.9198
8.2	9.9076
8.3	9.8956
8.4	9.8840
8.5	9.8726

**Table 4:** The variation of shock strength with shock front angle

Shock Front Angle $\theta$	Cos $\theta$	Shock strength (M)
0	1	10
15°	0.9659	3.1539
20°	0.9397	2.4437
25°	0.9063	2.0014
30°	0.866	1.704
35°	0.8192	1.4926
40°	0.766	1.3353
45°	0.707	1.215
50°	0.6428	1.1208
55°	0.5736	1.0453
60°	0.5	0.984
65°	0.423	0.9336
70°	0.342	0.8913

It is found that changing value of g increases 9.62 to 9.82, the shock strength takes the values from 4.1079 to 14.9842. It shows that as 'g' increases, shock strength M increases and changing value of r from 2.10 to 2.20, the shock strength takes the values from 2.3715 to 10.0000. It shows that as propagation distance (r) increases, shock strength (M) increases and changing the value of n from 7.3 to 8.5, the shock strength takes the values from 10.0296 to 9.8726 which shows that as specific heat index of water (n) increases, shock strength (M) decreases. The variation of shock strength with angle cos is computed. It is found that changing the values from 00 to 700, the shock strength takes the values from 10.0000 to 0.8913. It shows that as increases, shock strength (M) decreases.

**Variation of shock velocity with gravitational force per unit mass, propagation distance, specific heat index of water and angle -**

Expression (7) represents the shock velocity of spherical shock wave propagating freely in non-uniform region of water. It is found that shock velocity is a function of propagation distance r, specific heat index of water n, density  $\rho$ , angle cos  $\theta$  and force per unit mass i.e. Gravity. Initially taking M=10 at r=2.2 for n=7.5,  $\rho=1.22$ ,  $\rho = 3.684$  and, we have computed shock strength using equation (7) and obtained results are presented in tables (5), (6), (7) and (8).

**Table 5:** The variation of shock velocity with gravitational force per unit mass i.e. Gravity

Gravity (g)	Shock velocity (U)
9.62	6.1837
9.64	6.1796
9.66	6.1756
9.68	6.1715
9.70	6.1674
9.72	6.1633
9.74	6.1592
9.76	6.1551
9.78	6.1510
9.80	6.1469
9.82	6.1428

**Table 6:** The variation of shock velocity with propagation distance

Propagation distance (r)	Shock velocity (U)
2.1	6.6516
2.11	6.6007
2.12	6.5498
2.13	6.4991
2.14	6.4485
2.15	6.398
2.16	6.3476
2.17	6.2973
2.18	6.2471
2.19	6.197
2.2	6.1469

**Table 7:** The variation of shock velocity with specific heat index of water

Specific heat index of water (n)	Shock velocity (U)
7.3	6.0824
7.4	6.1147
7.5	6.1469
7.6	6.1790
7.7	6.2108
7.8	6.2426
7.9	6.2742
8.0	6.3056
8.1	6.3369
8.2	6.3680
8.3	6.3990
8.4	6.4299
8.5	6.4606

**Table 8:** The variation of shock velocity with shock front angle

1	Cos $\theta$	Shock velocity (U)
0	1	6.1469
15°	0.9659	6.2151
20°	0.9397	6.2669
25°	0.9063	6.3323
30°	0.866	6.4104
35°	0.8192	6.5
40°	0.766	6.6002
45°	0.707	6.7097
50°	0.6428	6.8268
55°	0.5736	6.9509
60°	0.5	7.0804
65°	0.423	7.2134
70°	0.342	7.3508

It is found that changing value of g increases 9.62 to 9.82, the shock velocity takes the values from 6.1837 to 6.1428. It shows that as 'g' increases, shock velocity U decreases and changing value of r from 2.10 to 2.20, the shock velocity takes the values from 6.6516 to 6.1469. It shows that as propagation distance (r) increases, shock velocity (U) decreases and changing the value of n from 7.3 to 8.5, the shock velocity takes the values from 6.0824 to 6.4606 which shows that as specific heat index of water (n) increases, shock velocity (U) increases. The variation of shock velocity with angle cos is computed. It is found that changing the values from 00 to 700,

the shock velocity takes the values from 6.1469 to 7.3508. It shows that as increases, shock velocity (U) increases.

**Variation of particle velocity with gravitational force per unit mass, propagation distance, specific heat index of water and angle**

Expression (8) represents the particle velocity of spherical shock wave propagating freely in non-uniform region of water. It is found that particle velocity is a function of propagation distance r, specific heat index of water n, density  $\rho$ , angle  $\cos \theta$  and force per unit mass i.e. Gravity.

Initially taking  $M=10$  at  $\alpha=2.2$  for  $n=7.5$ ,  $x=1.22$ ,  $\rho = 3.684$  and, we have computed particle velocity using equation (8) and obtained results are presented in tables (9), (10), (11) and (12).

**Table 9:** The variation of particle velocity with gravitational force per unit mass i.e. Gravity

Gravity (g)	Particle velocity (u)
9.62	1.4550
9.64	1.4540
9.66	1.4530
9.68	1.4521
9.70	1.4511
9.72	1.4502
9.74	1.4492
9.76	1.4482
9.78	1.4473
9.80	1.4463
9.82	1.4453

**Table 10:** The variation of particle velocity with propagation distance

Propagation distance (r)	Particle velocity (u)
2.1	1.565
2.11	1.5531
2.12	1.5411
2.13	1.5292
2.14	1.5173
2.15	1.5054
2.16	1.4935
2.17	1.4817
2.18	1.4699
2.19	1.4581
2.2	1.4463

**Table 11:** The variation of particle velocity with specific heat index of water

Specific heat index of water (n)	Particle velocity (u)
7.3	1.4656
7.4	1.4559
7.5	1.4463
7.6	1.4369
7.7	1.4277
7.8	1.4187
7.9	1.4099
8.0	1.4012
8.1	1.3927
8.2	1.3843
8.3	1.3761
8.4	1.3680
8.5	1.3601

**Table 12:** The variation of particle velocity with shock front angle

Shock Front Angle $q$	Cos $\theta$	particle velocity (u)
0	1	1.4463
15°	0.9659	1.4623
20°	0.9397	1.4745
25°	0.9063	1.4899
30°	0.866	1.5083
35°	0.8192	1.5294
40°	0.766	1.553
45°	0.707	1.5787
50°	0.6428	1.6063
55°	0.5736	1.6355
60°	0.5	1.6659
65°	0.423	1.6972
70°	0.342	1.7296

It is found that changing value of g increases 9.62 to 9.82, the particle velocity takes the values from 1.4550 to 1.4453. It shows that as 'g' increases, particle velocity u decreases and changing value of r from 2.10 to 2.20, the particle velocity takes the values from 1.5650 to 1.4463. It shows that as propagation distance (r) increases, particle velocity (u) decreases and changing the value of n from 7.3 to 8.5, the

particle velocity takes the values from 1.4656 to 1.3601 which shows that as specific heat index of water (n) increases, particle velocity (u) decreases. The variation of particle velocity with angle cos is computed. It is found that changing the values from 00 to 700, the particle velocity takes the values from 1.4463 to 1.7296. It shows that as increases, particle velocity (u) increases.

**Variation of pressure with gravitational force per unit mass, propagation distance, specific heat index of water and angle -**

Expression (9) represents the pressure of spherical shock wave propagating freely in non-uniform region of water. It is found that pressure is a function of propagation distance  $r$ , specific

heat index of water  $n$ , density  $\rho$ , angle  $\theta$  and force per unit mass i.e. Gravity.

Initially taking  $M=10$  at  $=2.2$  for  $n=7.5$ ,  $x=1.22$ ,  $= 3.684$  and, we have computed pressure using equation (9) and obtained results are presented in tables (13), (14), (15) and (16).

**Table 13:** The variation of pressure with gravitational force per unit mass i.e. Gravity

Gravity (g)	Pressure (P)
9.62	33.1464
9.64	33.1027
9.66	33.0590
9.68	33.0153
9.70	32.9716
9.72	32.9280
9.74	32.8843
9.76	32.8406
9.78	32.7969
9.80	32.7532
9.82	32.7095

**Table 14:** The variation of pressure with propagation distance

Propagation distance (r)	Pressure (P)
2.1	37.0821
2.11	36.6412
2.12	36.2022
2.13	35.7651
2.14	35.3297
2.15	34.8961
2.16	34.4643
2.17	34.0341
2.18	33.6055
2.19	33.1786
2.2	32.7532

**Table 15:** The variation of pressure with specific heat index of water

Specific heat index of water (n)	Pressure (P)
7.3	32.8418
7.4	32.7969
7.5	32.7532
7.6	32.7106
7.7	32.6692
7.8	32.6287
7.9	32.5894
8.0	32.5510
8.1	32.5135
8.2	32.4770
8.3	32.4413
8.4	32.4065
8.5	32.3725

**Table 16:** The variation of pressure with shock front angle

Shock Front Angle $\theta$	Cos $\theta$	Pressure (P)
0	1	32.7532
15°	0.9659	33.4831
20°	0.9397	34.044
25°	0.9063	34.7589
30°	0.866	35.6216
35°	0.8192	36.6234
40°	0.766	37.7622
45°	0.707	39.0251
50°	0.6428	40.3994
55°	0.5736	41.8807
60°	0.5	43.4561
65°	0.423	45.1044
70°	0.342	46.8382

It is found that changing value of  $g$  increases 9.62 to 9.82, the pressure takes the values from 33.1464 to 32.7095. It shows that as 'g' increases, pressure  $P$  decreases and changing value of  $r$  from 2.10 to 2.20, the pressure takes the values from 37.0821 to 32.7532. It shows that as propagation distance ( $r$ ) increases, pressure ( $P$ ) decreases and changing the value of  $n$  from 7.3 to 8.5, the pressure takes the values from 32.8418 to

32.3725 which shows that as specific heat index of water ( $n$ ) increases, pressure ( $P$ ) decreases. The variation of pressure with angle  $\cos$  is computed. It is found that changing the values from 0 to 70, the pressure takes the values from 32.7532 to 46.8382. It shows that as increases, shock pressure ( $P$ ) increases.

## Conclusions

It is concluded that shock strength (i.e. strength of tsunami) is depends on distance moves by tsunami, specific heat index of water 'n', density of water and earth gravity. It is found that-

1. Shock strength increases as gravity of earth increases.
2. Shock velocity decreases with increase in gravity.
3. The pressure and particle velocity behind the shock front decreases with the increases in gravitational force.
4. Strength of tsunami increases as it moves from it sources.
5. Shock velocity decreases as shock advances.
6. The pressure and particle velocity both decreases with propagation distance.
7. Strength of tsunami decreases as specific heat index of water increases.
8. The pressure and particle velocity decreases as specific heat index of water increases.
9. Strength of tsunami decreases as angle  $\theta$  increases.
10. Shock velocity, the pressure and particle velocity increase with angle  $\theta$ .

## References

1. Thomas TY. J Math. 1957; 6:607-620.
2. Bhutani OP. J Math. Anal. Applic. 1966; 13:565-576.
3. Singh VP, Bola MS. Ind. J Pure Appl. 1976; 7(12):1405-1410.
4. Singh VP, Madan AK, Suneja SR, Lal Proc. Ind. Acad. Sci., Engg. Sci. 1980; 3:169-175.
5. Singh VP. Def. Sci., J. 1982; 32:327-332.
6. Vishvakarma JP, Nagar KS, Mishra RB. Def. Sci. J. 1988; 38(1):69-76.
7. Yadav RP. Mod. Meas Cont., B. 1992; 46(4)1.
8. Yadav RP, Tripathi S. Astro. Phy. Spaced, Sci. 1995; 225:67.
9. Terao, Akaba H, Shiraishi H. Shock Waves. 1995; 4:187.
10. Mazell P, Saurel R, Laured JC, Butler P B. Combust Flame. 1996; 113:1.
11. Li H, Ben-Dor G. Combust Flame. 1998; 113:1.
12. Ishii R, Fujimoto H, Hatta N. Phys. Fluid. 1999; 11(7):1921
13. Sacito T, Marumoto M, Yamashita SHR, Nakagawa A, Hirano T, Takayama K. Shock Waves. 2003; 13(2):139.
14. Yadav RP, Gangwar PK. J Nat. Phy. Sci. 2003; 17(2):109.
15. Chester W. Philos. Mag. 1954; 45(7):1293
16. Chisnell RF. J Fluid. Mech. 1957; 854:375.
17. Witham GB. J Fluid Mech. 1958; 4:337.
18. Yadav RP, Sharma Rahul. Act. Cin. Ind. 2004; 31(1):095.
19. Yadav RP, Kumar A, Pal V, Kumar K. International, J Maths. Sci. Eng. Appl. 2010; 4(5):45.