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On the non-homogeneous ternary cubic equation $3(x^2+y^2)-5xy=47z^3$

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Abstract
The non-homogeneous cubic equation with three unknowns represented by $3(x^2+y^2)-5xy=47z^3$ is analysed for its patterns of non-zero distinct integer solutions. A few interesting relations among the solutions are presented.

Keywords: non-homogeneous cubic, ternary cubic, integer solutions

Introduction
It is well known that the Diophantine equations are rich in variety [1-3]. In particular, one may refer [4-13] for cubic with three unknowns. In this paper yet another cubic equation with three unknowns is given by $3(x^2+y^2)-5xy=47z^3$ is considered for determining its infinitely many non-zero integer solutions. Also, A few interesting relations among the solutions are exhibited.

Method of analysis
The non-homogeneous cubic equation to be solved is

$$3(x^2+y^2)-5xy=47z^3 \tag{1}$$

Introducing the linear transformations

$$x=u+v,\ y=u-v \tag{2}$$

in (1),it gives

$$u^2+11v^2=47z^3 \tag{3}$$

Assume

$$z=z(a,b)=a^2+11b^2 \tag{4}$$

Also write 47 as

$$47=(6+i\sqrt{11})(6-i\sqrt{11}) \tag{5}$$

Substituting (4), (5) in (3) and applying the method of factorization,we have

$$(u+i\sqrt{11}v)(u-i\sqrt{11}.v)=(6+i\sqrt{11})(6-i\sqrt{11})(a+i\sqrt{11}b)^3(a-i\sqrt{11}b)^3$$

Equating the positive and negative terms in the above equation, we have

$$(u+i\sqrt{11}v)=(6+i\sqrt{11})(a+i\sqrt{11}b)^3 \tag{6}$$

$$\left(u - i\sqrt{11}\,v\right) = \left(6 - i\sqrt{11}\right)\left(a - i\sqrt{11}\,b\right)^3 \tag{7}$$

Equating the real and imaginary parts in either (6) or (7), we have

$$u = 6a^3 - 198\,ab^2 - 33\,a^2 b + 121\,b^3$$

$$v = 18\,a^2 b - 66\,b^3 + a^3 - 33\,ab^2$$

Substituting the values of u and v in (2), we get]

$$x = x(a,b) = 7a^3 - 231\,ab^2 - 15\,a^2 b + 55\,b^3 \tag{8}$$

$$y = y(a,b) = 5a^3 - 165\,ab^2 - 51\,a^2 b + 187\,b^3 \tag{9}$$

Thus, (4), (8) and (9) represent the integer solutions to (1).
Properties:

1. $x(a,a) + 184\ CP_{a,6} = 0$
2. $y(a,1) - 5CP_{a,6} + 51\,PR_a + 57\,GNO_a \equiv 0 \pmod{2}$
3. $z(a,1) - PR_a + a \equiv 0 \pmod{11}$

However, we have other choices of integer solutions to (1) that are illustrated below:
(3) is written as

$$u^2 + 11v^2 = 47\,z^3 * 1 \tag{10}$$

Assume $1 = \dfrac{\left(1 + i\,3\sqrt{11}\right)\left(.1 - i\,3\sqrt{11}\right)}{100}$ (11)

Substituting (4), (5) and (11) in (10) and employing the method of factorization, define

$$\left(u + i\sqrt{11}\,v\right)\left(u - i\sqrt{11}\,.v\right) = \frac{\left(6 + i\sqrt{11}\right)\left(6 - i\sqrt{11}\right)\left(1 + i\,3\sqrt{11}\right)\left(1 - i\,3\sqrt{11}\right)\left(a + i\sqrt{11}\,b\right)^3\left(a - i\sqrt{11}\,b\right)^3}{100}$$

Equating the positive and negative terms in the above equation, we have

$$\left(u + i\sqrt{11}\,v\right) = \frac{1}{10}\left(6 + i\sqrt{11}\right)\left(a + i\sqrt{11}\,b\right)^3\left(1 + i\,3\sqrt{11}\right) \tag{12}$$

$$\left(u - i\sqrt{11}\,v\right) = \frac{1}{10}\left(6 - i\sqrt{11}\right)\left(a - i\sqrt{11}\,b\right)^3\left(1 - i\,3\sqrt{11}\right) \tag{13}$$

Equating the real and imaginary parts in either (12) or (13), we have

$$u = \frac{1}{10}\left[-27\,a^3 + 891\,ab^2 - 627\,a^2 b + 2299\,b^3\right]$$

$$v = \frac{1}{10}\left[19\,a^3 - 627\,ab^2 - 81\,a^2 b + 297\,b^3\right]$$

Substituting the above values of u and v in (2) and replacing a by 10A and b by 10B, we get

$$x = x(A,B) = \left[-800A^3 + 259600B^3 + 26400AB^2 - 70800A^2B\right] \quad (14)$$

$$y = y(A,B) = \left[-4600A^3 + 200200B^3 + 151800AB^2 - 54600A^2B\right] \quad (15)$$

And from (4)

$$z = z(A,B) = 100A^2 + 1100B^2 \quad (16)$$

Thus, (14), (15) and (16) represents the integer solutions to (1)
Properties:

1. $x(1,B) - 129800SO_B - 26400PR_B + 84220B \equiv 0 \pmod 2$
2. $y(B,B) - 292800CP_{B,6} = 0$
3. $Z(A,A) - 1200PR_A + 600GNO_A + 600 = 0$

Note: It is to be noted that, in addition to (11), 1 may also be represented as

$$1 = \frac{(7 + i5\sqrt{11})(7 - i5\sqrt{11})}{324} \quad (17)$$

Further, considering (5) with (17) and performing a few calculations the integer solutions of (1) are given by

$$x = x(A,B) = \left[7776A^3 + 1496880B^3 - 256608AB^2 - 408240A^2B\right]$$

$$y = y(A,B) = \left[-16200A^3 + 1404216B^3 + 534600AB^2 - 382968A^2B\right]$$

$$z = z(A,B) = 324A^2 + 3564B^2$$

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