International Journal of Academic Research and Development
ISSN: 2455-4197, Impact Factor: RJIF 5.22
www.academicsjournal.com
Volume 1; Issue 11; November 2016; Page No. 29-31

Special pairs of Pythagorean triangles and 3-digit consecutive sphenic numbers

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Abstract

We present pairs of Pythagorean triangles such that in each pair, the difference between their perimeters is six times the 3-digit consecutive Sphenic numbers 230 and 231. Also we present the number of pairs of primitive and non-primitive Pythagorean triangles.

Keywords: pythagorean triangles, 3-digit sphenic numbers, primitive and non-primitive pythagorean triangles

1. Introduction

Number theory is a broad and diverse part of Mathematics that developed from the study of the integers. Mathematicians all over the ages have been fascinated by Pythagorean Theorem and are solving many problems related to it thereby developing Mathematics. Pythagorean triangles which were first studied by the Pythagoreans around 400 B.C., remains one of the fascinated topics for those who just adore the numbers.

A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. These numbers have presented in [1-5]. In [6-14], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are presented.

In this communication, we search for pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is six times the 3-digit consecutive sphenic numbers 230 and 231.

2. Basic Definitions

2.1 Definition

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$. Also, in Pythagorean triangle $T(x, y, z): x^2 + y^2 + z^2$, x and y are called its legs and z its hypotenuse.

2.2 Definition

The most cited solutions of the Pythagorean equation is $x = m^2 - n^2,\ y = 2mn,\ z = m^2 + n^2$, where $m > n > 0.$ This solution is called primitive, if m, n are of opposite parity and gcd $(m, n) = 1.$

2.3 Definition

Sphenic number is a positive integer which is the product of three distinct prime numbers.

3. Method of Analysis

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators $m, q\ (m > q > 0)$ and $p, q\ (p > q > 0)$ respectively.

Let P_1, P_2 be the perimeters of PT_1 & PT_2 such that $P_1 - P_2$ = 6 times the consecutive 3-digit sphenic numbers 230 and 231.

The above relation leads to the equation

$$(2m + q)^2 - (2p + q)^2 = 230 \quad (1)$$

Which simplifies to

$$(m - p)(m + p + q) = 690 \quad (2)$$

After completing the numerical computations, it is noted that there are 266 values of m, p & q satisfying (2).

We have presented the values of m, p, q, P_1, P_2 in the Table 1 as below:

Table 1

S. No.	m	p	q	P_1	P_2	$\frac{P_1 - P_2}{6}$
1	345	344	1	238740	237360	230
2	344	343	3	238736	237356	230
3	343	342	5	238728	237348	230
4	342	341	7	238716	237336	230
5	341	340	9	238700	237320	230
6	340	339	11	238680	237300	230
7	339	338	13	238656	237276	230
8	338	337	15	238628	237248	230
9	337	336	17	238596	237216	230
10	336	335	19	238560	237180	230
11	335	334	21	238520	237140	230
12	334	333	23	238476	237096	230
13	333	332	25	238428	237048	230
14	332	331	27	238376	236996	230
15	331	330	29	238320	236940	230
16	330	329	31	238260	236880	230
17	329	328	33	238196	236816	230
18	328	327	35	238128	236748	230
19	327	326	37	238056	236676	230
20	326	325	39	237980	236600	230
⋮	⋮	⋮	⋮	⋮	⋮	⋮
247	46	40	29	6900	5520	230
248	45	39	31	6840	5460	230
249	44	38	33	6776	5396	230
250	43	37	35	6708	5328	230
251	39	29	1	3120	1740	230
252	38	28	3	3116	1736	230
253	37	27	5	3108	1728	230
254	36	26	7	3096	1716	230
255	35	25	9	3080	1700	230
256	34	24	11	3060	1680	230
257	33	23	13	3036	1656	230
258	32	22	15	2976	1628	230
259	31	21	17	2940	1596	230
260	30	20	19	1860	1560	230
261	30	15	1	1856	480	230
262	29	14	3	1848	476	230
263	28	13	5	1836	468	230
264	27	12	7	1820	456	230
265	26	11	9	230	440	230
266	26	3	1	230	24	230

Hence there are 266 pairs of Pythagorean triangles, out of which there are 39 primitive pairs of Pythagorean triangles and 49 non-primitive pairs of Pythagorean triangles and in the remaining 178 pairs, one is primitive and the other is non-primitive Pythagorean triangles.
A similar observation is observed regarding other 3-digit sphenic number 231 and is exhibited in the table 2 below:

Table 2

S. No.	m	p	q	P_1	P_2	$\frac{P_1 - P_2}{6}$
1	346	345	2	240816	239430	231
2	345	344	4	240810	239424	231
3	344	343	6	240800	239414	231
4	343	342	8	240786	239400	231
5	342	341	10	240768	239382	231
6	341	340	12	240746	239360	231
7	340	339	14	240720	239334	231
8	339	338	16	240690	239304	231
9	338	337	18	240656	239270	231
10	337	336	20	240618	239232	231

⋮	⋮	⋮	⋮	⋮	⋮	⋮
177	32	23	22	3456	2070	231
178	36	25	2	2736	1350	231
179	35	24	4	2730	1344	231
180	34	23	6	2720	1334	231
181	33	22	8	2706	1320	231
182	32	21	10	2688	1302	231
183	31	20	12	2666	1280	231
184	30	19	14	2640	1254	231
185	29	18	16	2610	1224	231
186	26	5	2	1456	70	231

Hence there are 186 pairs of Pythagorean triangles, out of which there are 12 non-primitive pairs of Pythagorean triangles and in the remaining 174 pairs, one is primitive and the other is non-primitive Pythagorean triangles.

4. Conclusion
In this communication, it is observed that there are only finitely many Pythagorean triangles satisfying the property under consideration. The total number of pairs of primitive and non-primitive Pythagorean triangles are also given.
To conclude, one may search for the connections between the pairs of Pythagorean triangles and other higher order sphenic numbers.

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